

Rachunek Prawdopodobieństwa 2

Zestaw zadań nr 4

Termin realizacji: 14 XI 2008

1. For a simple random walk S with $S_0 = 0$ and $p < 1/2$, show that the maximum $M = \max\{S_n : n \geq 0\}$ satisfies $P(M \geq r) = (p/q)^r$ for $r \geq 0$.
2. For a simple random walk S with $S_0 = 0$, let T_b be the number of steps until the walk first reaches b where $b > 0$. Show that $E(T_b | T_b < \infty) = b/|p - q|$.
3. Pokazać, że dla każdego $m \geq 1$,

$$\frac{2}{m} \binom{2m-2}{m-1} 2^{-2m} = (-1)^{n+1} \binom{\frac{1}{2}}{m}.$$

4. Pokazać, że

$$\frac{2}{t^2} \left(\sqrt{1 - s^2 t^2} - \sqrt{1 - t^2} \right) = \frac{1}{n+1} \sum_{n=0}^{\infty} t^{2n} P(S_{2n} = 0) (1 - s^{2n+2}).$$

5. Let (Z_n) , $n \geq 0$, be a branching process with $Z_0 = 1$. Find an expression for the generating function G_n of Z_n if $G_1 = 1 - \alpha(1 - s)^\beta$, $0 < \alpha, \beta < 1$.
6. Each generation of a branching process is augmented by a random number of immigrants who are indistinguishable from the other members of the population. Suppose that the numbers of immigrants in different generations are independent of each other and of the past history of the branching process, each such number having probability generating function (pgf) $H(s)$. Show that the pgf G_n of the size of the n th generation satisfies $G_{n+1}(s) = g_n(G(s))H(s)$, where G is the pgf of a typical family of offsprings.